NO CALCULATORS:

1. Find $\lim_{x\to\infty}\frac{7x^3+6x-3}{7x^2+9}$. degree on top > degree on bottom

A) 1 B) 0 C) $-\infty$ D) ∞

2. Find the points of discontinuity of the function $y=\frac{x^2+9x+20}{x^2-25}$. For each discontinuity identify the type of discontinuity. $\frac{(x+4)(x+5)}{(x+5)(x-5)}$ $x=5$ V.A (Inf. disc.) $x=-5$ Hole (removable disc.)

A. Removable discontinuity at $x=7$
Removable discontinuity at $x=-7$

B. Removable discontinuity at $x=-5$
Infinite discontinuity at $x=5$

C. Removable discontinuity at $x=-5$
Infinite discontinuity at $x=7$

D. Oscillating discontinuity at $x=5$
Removable discontinuity at $x=-5$

3. Find y'' if $y=2x^4+7x^3-\frac{1}{2}x^2-24x-14$. $y'=8x^3+21x^2-x-24$ $y''=24x^2+42x-1$

A) $y''=24x^2+42x-1$ B) $y''=24x^2+42x-24$ C) $y''=8x^2+21x-1$ D) $y''=8x^2+21x-24$

4. Suppose *u* and *v* are differentiable at $x=6$, and that $u(6)=5$, $v(6)=-2$, $u'(6)=-8$, and $v'(6)=9$. Find $\frac{d}{dx}(\frac{u}{v})$ at $x=6$. Need some space, at end!

A) $-\frac{29}{4}$ B) $\frac{61}{4}$ C) $-\frac{4}{29}$ D) $\frac{20}{9}$

5. Find $\frac{dy}{dx}$ if $y=(3x+5)^8$. $y'=8(3x+5)^7\cdot 3=24(3x+5)^7$

A) $\frac{dy}{dx}=24(3x+5)^7$ B) $\frac{dy}{dx}=24(3x+5)^8$ C) $\frac{dy}{dx}=8(3x+5)^7$ D) $\frac{dy}{dx}=24x^7(3x+5)$

6. Find $\frac{dy}{dx}$ if $y=\tan^{-1}(3x)$. $y'=\frac{1}{1+(3x)^2}\cdot 3=\frac{3}{1+9x^2}$

A) $\frac{dy}{dx}=\frac{1}{1+9x^2}$ B) $\frac{dy}{dx}=\frac{3}{1+9x^2}$ C) $\frac{dy}{dx}=\frac{-3}{1+9x^2}$ D) $\frac{dy}{dx}=\frac{3x}{1+9x^2}$

7. Find $\frac{dy}{dx}$ if $y=3^{-5x}$. $\ln y=\ln(3^{-5x})$ $\ln y=-5x\ln 3$ $\frac{1}{y}\frac{dy}{dx}=-5\ln 3$

A) $\frac{dy}{dx}=-15\ln(3)$ B) $\frac{dy}{dx}=3^{-5x}\ln(3)$ C) $\frac{dy}{dx}=-5\bullet 3^{-5x}\ln(3)$ D) $\frac{dy}{dx}=-15^{-5x}\ln(3)$

$\frac{dy}{dx}=-5y\ln 3=-5\cdot 3^{-5x}\ln 3$

$f(3)=-12$ $f(6)=-87$ $\frac{-87+12}{6-3}=-25$

8. If $f(x)=-3x^2+2x+9$ is continuous on $[3,6]$ and differentiable on $(3,6)$, then, according to the Mean Value Theorem, there is at least one point *c* in $(3,6)$ at which _____.

A) $f'(c)=-6$ B) $f(c)=-6$ C) $f(c)=-25$ D) $f'(c)=-25$

9. Find the function whose derivative is $f'(x)=6x+2$ and whose graph passes through the point $P(0,-12)$. $f(x)=3x^2+2x+C$ $f(0)=C=-12$ $f(x)=3x^2+2x-12$

A) $f(x)=6$ B) $f(x)=3x^2+2x$ C) $f(x)=6x-12$ D) $f(x)=3x^2+2x-12$

10. Find the linearization $L(x)$ of $f(x)=-2x^2-5x+17$ at $x=-2$. $f'(x)=-4x-5$ $f'(-2)=3$

A) $L(x)=x-2$ B) $L(x)=3x+25$ C) $L(x)=3x+17$ D) $L(x)=3x+19$

$y-19=3(x+2)$ $y=3x+6+19$ $y=3x+25$ $f(-2)=19$

11. Express $\lim\limits_{\|P\|\to 0}\sum\limits_{k=1}^{n}(4c_k^3+3c_k^2-7)\Delta x_k$ as a definite integral on the interval $[4,6]$.

A) $\int_4^6(12x^2+6x-7)dx$ B) $\int_4^6(4x^3+3x^2-7)dx$ C) $\int_4^6(12x^2+6x)dx$ D) $\int_6^4(4x^3+3x^2-7)dx$

12. Use the Fundamental Theorem of Calculus with the chain rule to find $\frac{dy}{dx}$ if $y=\int_4^{3x}(7-7t)dt$.

At end, to show some work!

A) $\frac{dy}{dx}=-63x$ B) $\frac{dy}{dx}=21-63x$ C) $\frac{dy}{dx}=21-21x$ D) $\frac{dy}{dx}=7-7x$

13. Evaluate the integral $\int_1^3 x^3\ln x\,dx$. Need space, at end!

A) $\frac{81}{4}\ln 3-4$ B) $27\ln 3-\frac{20}{3}$ C) $\frac{81}{4}\ln 3+9$ D) $\frac{81}{4}\ln 3-5$

14. Which of the following integrals gives the length of the curve $y=\cos 2x$ from $x=0$ to $x=5$?

$y'=-2\sin(2x)$ $(y')^2=4\sin^2(2x)$

A) $\int_0^5\sqrt{1+2\cos 2x}\,dx$ B) $\int_0^5\sqrt{1-4\cos^2 2x}\,dx$ C) $\int_0^5\sqrt{1+4\sin^2 2x}\,dx$ D) $\int_0^5\sqrt{1+2\sin 2x}\,dx$

15. Evaluate the integral $\int 2\sec t\tan t\,dt$. $=2\sec t+C$

A) $\sec^2 t+C$ B) $2\sec t+C$ C) $\sec t+C$ D) $2\tan^2 t+C$

16. Let $f(x)=x^4+ax^2$. What is the value of *a* if *f* has a local minimum at $x=-6$?

A) $a=-72$ B) $a=0$ C) $a=-216$ D) $a=72$

$f'(x)=4x^3+2ax=0$ $f'(-6)=-864-12a=0$

$f''(x)=12x^2+2a>0$ $a=-72$

17. Evaluate the integral $\int \ln(3x)dx$

$u=\ln(3x)$ $dv=dx$ $du=\frac{1}{x}dx$ $v=x$ $x\ln(3x)-\int dx$ $x\ln(3x)-x+C$

A) $x\ln 3x - x + C$ B) $\frac{1}{x}+C$ C) $\frac{1}{3x}+C$ D) $3x\ln 3x - 3x + C$

18. For which of the following differential equations will a slope field show nothing but negative slopes in the fourth quadrant?

$x>0$ $y<0$

A) $\frac{dy}{dx} = -\frac{x}{y}$ B) $\frac{dy}{dx} = xy + 5$ C) $\frac{dy}{dx} = \frac{y}{x^2} - 3$ D) $\frac{dy}{dx} = xy^2 - 2$

19. **(calculator OK on this one)** Use Euler's Method with $\Delta x = 0.1$, $\frac{dy}{dx} = 2x - y$ and $y = 0$ when $x = 1$ to find the value of y when $x = 1.3$.

A) 0.6 B) 0.2 C) 0.4 D) 0.8

20. Find y if $\frac{dy}{dx} = 2xy$ and $y = 1$ when $x = 0$

$\frac{1}{y}dy = 2x\,dx$ $\ln|y| = x^2 + C$ $y = e^{x^2+C}$

A) y^{2x} B) e^{x^2} C) $x^2 y$ D) $\frac{x^2y^2}{2}+1$

21. **(calculator OK on this one)** The logistic differential equation $\frac{dP}{dt} = 0.04P(90 - P)$ describes the growth of a population *P*, where *t* is measured in years. Find the rate at which the population is growing when it is growing the fastest.

A) 81 B) 90 C) 45 D) .04

CALCULATORS OK:

wor at end, need space!

22. A tanker is spilling oil into the water resulting in an oil slick that is close to circular. At the time that the slick's diameter is growing at the rate of 7m/sec, the diameter is 200 meters. At what rate is the area of the oil slick increasing?

A) 700.000 $m^2/\sec$ B) 4398.230 $m^2/\sec$ C) 314.159 $m^2/\sec$ D) 2199.115 $m^2/\sec$

23. Use $LRAM_4$ to compute the area under the curve described by $y = -2x + 9$ over the interval $0 \le x \le 2$.

A) 15 B) -1.5 C) 4.5 D) 17.5

24. Find the average value of the function $y = -x^2 + 4x + 14$ over the interval $[2, 6]$.

$\frac{1}{4}\int_2^6 -x^2+4x+14\,dx$

A) -12.67 B) 26.67 C) 50.67 D) 12.67

25. The function $v(t) = 16t^2 - 5t$ is the velocity in m/sec of a particle moving along the x-axis, where *t* is measured in seconds. Use analytic methods to find the particle's displacement for $0 \le t \le 7$. Round your answer to the nearest 1 m.

A) 5243 m B) 1707 m C) 219 m D) 2662 m

$\int_0^7 v(t)\,dt$

26. Let $f(x)=x^3+3x^2+5x-30$ and let *g* be the inverse function of *f*. What is the value of $g'(0)$?

A) $-\frac{1}{29}$ B) $\frac{1}{29}$ C) $\frac{1}{5}$ D) 5

27. Find the area of the region enclosed by $y=|x^2-25|$ and $y=\frac{x^2}{2}+25$.

$2\int_0^{10}\left(\frac{x^2}{2}+25-|x^2-25|\right)dx$

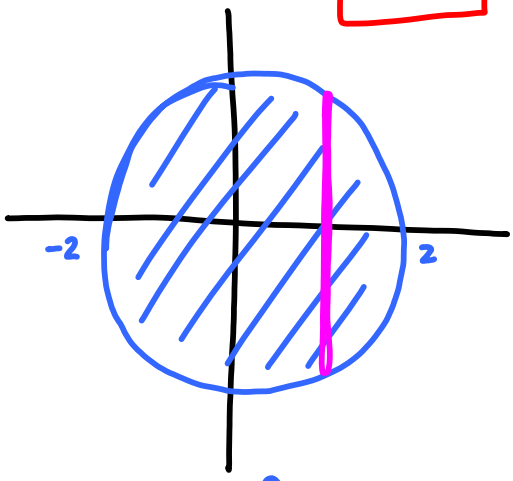
A) $\frac{500}{3}$ B) $\frac{250}{3}$ C) 1000 D) $\frac{1000}{3}$

28. A cup of coffee with temperature $104°$ F is placed in a freezer with temperature $0°$ F. After 6 minutes, the temperature of the coffee is $63.2°$ F. When will its temperature be $20°$ F ? Round your answer to the nearest minute.

A) 21 minutes after being placed in the freezer
B) 24 minutes after being placed in the freezer
C) 20 minutes after being placed in the freezer
D) 14 minutes after being placed in the freezer

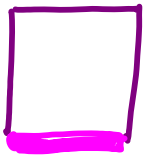
29. The solid lies between planes perpendicular to the x-axis at $x=-2$ and $x=2$. The cross sections perpendicular to the x-axis between these planes are squares whose bases run from the semicircle $y=-\sqrt{4-x^2}$ to the semicircle $y=\sqrt{4-x^2}$. Find the volume of the solid.

A) $\frac{64}{3}$ B) $\frac{128}{3}$ C) $\frac{44}{3}$ D) $\frac{88}{3}$



Base Region

Cross Sections



$A(x)=4(4-x^2)$

$s=2\sqrt{4-x^2}$

$V=2\int_0^2 4(4-x^2)\,dx=8\int_0^2 4-x^2\,dx$

$V=8\left(4x-\frac{x^3}{3}\right]_0^2\right)=8\left(8-\frac{8}{3}\right)=8\left(\frac{2\cdot 8}{3}\right)$

$V=128/3$

OPEN ENDED REVIEW: Answers at end!!!

1. Let f be the function defined by $f(x) = (x^2+1)e^{-x}$ for $-4 < x < 4$.

a) For what value of *x* does *f* reach its absolute maximum. Justify your answer.

b) Find the *x*-coordinates of all points of inflection of *f*. Justify your answer.

c) Sketch the graph.

2. A particle moves along the x-axis with an acceleration given by $a(t) = \cos t$ for $t > 0$. At $t = 0$, the velocity, $v(t)$, of the particle is 2 and the position $x(t)$ is 5.

a) Write an expression for the velocity, $v(t)$ of the particle.

b) Write an expression for the position $x(t)$.

c) For what values of *t* is the particle moving right? Justify your answer.

d) Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.

3. Let *R* be the region enclosed by the graphs of $y = e^x$, $y = x$, and the lines $x = 0$ and $x = 4$.

a) Find the area of region *R*.

b) Find the volume of the solid generated when *R* is revolved about the x-axis.

c) Set up an integral expression in terms of a single variable for the volume of the solid generated when *R* is revolved about the y-axis and then find the volume.

4. Suppose u and v are differentiable at $x=6$, and that $u(6)=5, v(6)=-2, u'(6)=-8$, and $v'(6)=9$.

Find $\frac{d}{dx}\left(\frac{u}{v}\right)$ at $x=6$.

A) $-\frac{29}{4}$ B) $\frac{61}{4}$ C) $-\frac{4}{29}$ D) $\frac{20}{9}$

$$\frac{\frac{du}{dx}\cdot v - u\cdot\frac{dv}{dx}}{v^2} = \frac{-8(-2)-5(9)}{(-2)^2} = \frac{16-45}{4} = -\frac{29}{4}$$

12. Use the Fundamental Theorem of Calculus with the chain rule to find $\frac{dy}{dx}$ if $y=\int_4^{3x}(7-7t)dt$.

A) $\frac{dy}{dx}=-63x$ B) $\frac{dy}{dx}=21-63x$ C) $\frac{dy}{dx}=21-21x$ D) $\frac{dy}{dx}=7-7x$

$$y=\int_4^{3x}(7-7t)\,dt = G(3x)-G(4) \quad \text{where } G'(x)=7-7x$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[G(3x)\right] = 3\cdot G'(3x) = 3\left(7-7(3x)\right) = 21-63x$$

13. Evaluate the integral $\int_1^3 x^3\ln x\,dx$.

A) $\frac{81}{4}\ln 3-4$ B) $27\ln 3-\frac{20}{3}$ C) $\frac{81}{4}\ln 3+9$ D) $\frac{81}{4}\ln 3-5$

$$\int x^3\ln x\,dx = \frac{x^4\ln x}{4} - \frac{1}{4}\int x^3\,dx = \frac{x^4\ln x}{4} - \frac{1}{16}x^4+C$$

$u=\ln x \qquad dv=x^3dx$

$du=\frac{1}{x}dx \qquad v=\frac{x^4}{4}$

$$\int_1^3 x^3\ln x\,dx = \frac{81\ln 3}{4} - \frac{81}{16}+\frac{1}{16} = \frac{81}{4}\ln 3-5$$

22. A tanker is spilling oil into the water resulting in an oil slick that is close to circular. At the time that the slick's diameter is growing at the rate of 7m/sec, the diameter is 200 meters. At what rate is the area of the oil slick increasing?

A) 700.000 m^2/sec B) 4398.230 m^2/sec C) 314.159 m^2/sec D) 2199.115 m^2/sec

$\frac{dd}{dt} = 7 m/sec$ when $d = 200 m$

$A = \pi\left(\frac{d}{2}\right)^2$

$A = \frac{\pi}{4} d^2$

$\frac{dA}{dt} = \frac{\pi}{2} d \frac{dd}{dt}$

$\frac{dA}{dt} = \frac{\pi}{2}(200)(7) = 700\pi \; m^2/sec$

26. Let $f(x) = x^3 + 3x^2 + 5x - 30$ and let g be the inverse function of f. What is the value of $g'(0)$?

A) $-\frac{1}{29}$ B) $\frac{1}{29}$ C) $\frac{1}{5}$ D) 5

$f(x) = x^3 + 3x^2 + 5x - 30 = 0$

$g'(0) = \frac{1}{f'(2)} = 1/29$

$f(2) = 8 + 3(4) + 5(2) - 30 = 0$

$f'(x) = 3x^2 + 6x + 5$

$f'(2) = 12 + 12 + 5 = 29$

28. A cup of coffee with temperature 104°F is placed in a freezer with temperature 0°F. After 6 minutes, the temperature of the coffee is 63.2°F. When will its temperature be 20°F ? Round your answer to the nearest minute.

A) 21 minutes after being placed in the freezer B) 24 minutes after being placed in the freezer

C) 20 minutes after being placed in the freezer D) 14 minutes after being placed in the freezer

$T_0 = 104°F$

$T_s = 0°F$

$t = 6$

$T(6) = 63.2°F$

$T - T_s = (T_0 - T_s)e^{-kt}$

$T(t) = 104\, e^{-kt}$

$T(6) = 104\, e^{-6k} = 63.2 \quad k \approx 0.0830$

$20 = 104\, e^{-0.0830t} \Rightarrow t \approx 19.860$

OPEN ENDED REVIEW:

1. Let f be the function defined by $f(x) = (x^2+1)e^{-x}$ for $-4 < x < 4$.

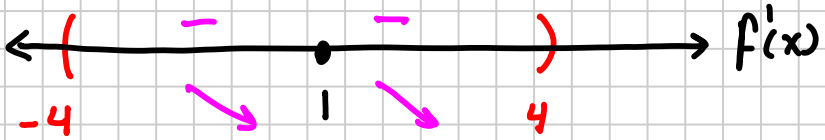
a) For what value of x does f reach its absolute maximum. Justify your answer.

b) Find the x-coordinates of all points of inflection of f. Justify your answer.

c) Sketch the graph.

a) $f'(x) = 2x e^{-x} + (x^2+1)e^{-x}(-1) = e^{-x}[2x-(x^2+1)]$

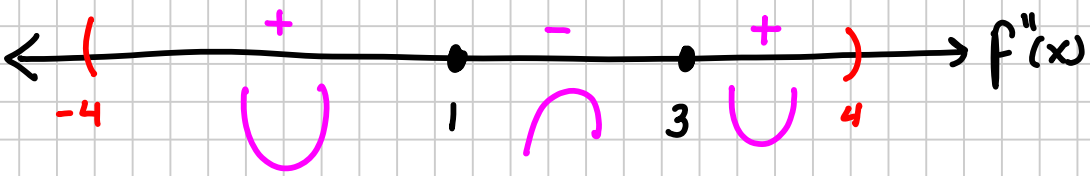
$f'(x) = -e^{-x}[x^2-2x+1] = -e^{-x}(x-1)^2 = 0 \Rightarrow x = 1$



f(x) does not reach an abs. max. f(x) is monotonic decreasing on an open interval ∴ does not reach an abs. max or abs. min.

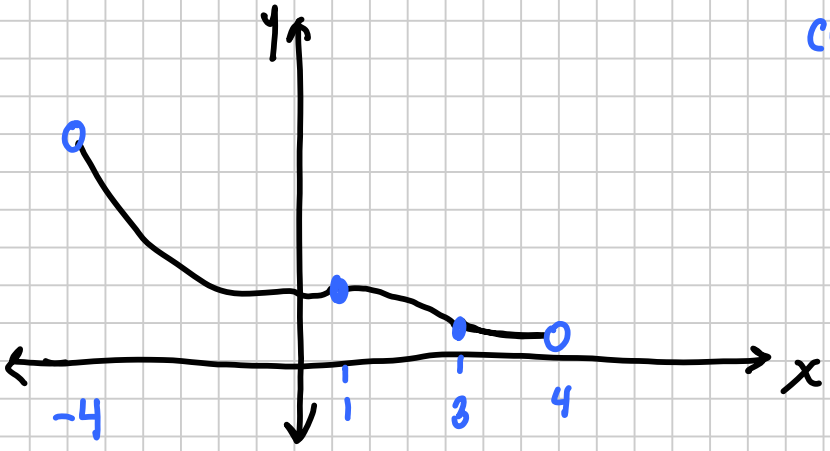
b) $f''(x) = e^{-x}(x-1)^2 - e^{-x}\cdot 2(x-1)$

$f''(x) = (x-1)e^{-x}[x-1-2] = (x-1)e^{-x}(x-3)$



POI: at $x=1$ and $x=3$

f(x) concave up: $(-4,1) \cup (3,4)$
concave down: $(1,3)$
} 2nd Deri. Test



2. A particle moves along the x-axis with an acceleration given by $a(t) = \cos t$ for $t > 0$. At $t = 0$, the velocity, $v(t)$, of the particle is 2 and the position $x(t)$ is 5.

a) Write an expression for the velocity, $v(t)$ of the particle.

b) Write an expression for the position $x(t)$.

c) For what values of t is the particle moving right? Justify your answer.

d) Find the total distance traveled by the particle from $t = 0$ to $t = \frac{\pi}{2}$.

a) $V(t) = \sin(t) + C$
$V(0) = C = 2$
$\Rightarrow$ $V(t) = \sin(t) + 2$

b) $x(t) = -\cos t + 2t + C$
$x(0) = -1 + C = 5$
$C = 6$
$\Rightarrow$ $x(t) = -\cos t + 2t + 6$

c) The particle is moving right, meaning $x(t)$ increasing, when $V(t)$, its derivative, is positive.

$V(t) > 0$
$\sin(t) + 2 > 0 \Rightarrow$ for all values of t

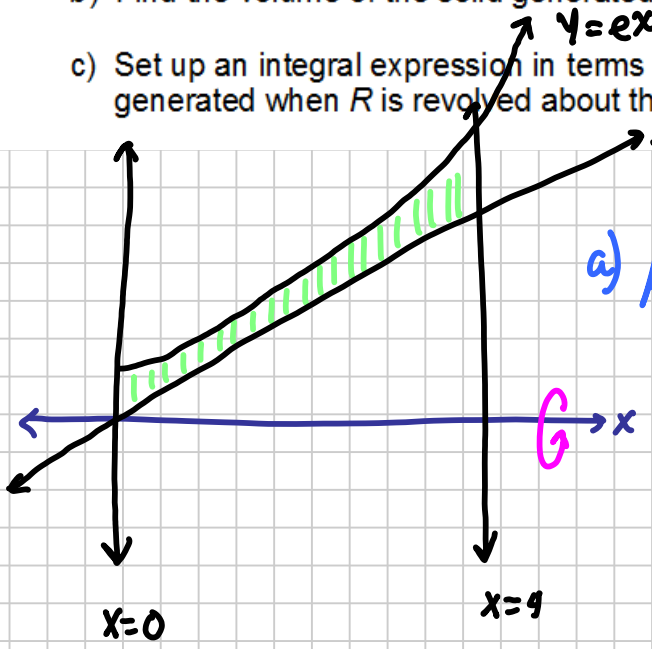
d) $x(\pi/2) - x(0) = -\cos(\pi/2) + 2(\pi/2) + 6 - (-\cos 0 + 2(0) + 6)$
$= \pi + 1$

3. Let R be the region enclosed by the graphs of $y = e^x$, $y = x$, and the lines $x = 0$ and $x = 4$.

a) Find the area of region R.

b) Find the volume of the solid generated when R is revolved about the x-axis.

c) Set up an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis and then find the volume.



a) $A = \int_0^4 e^x - x \, dx = e^x - \frac{x^2}{2} \Big]_0^4$

$= e^4 - 8 - 1$

$A = e^4 - 9$

b) $V = \pi \int_0^4 e^{2x} - x^2 \, dx \approx 4{,}613.886$

c) $V = 2\pi \int_0^4 x(e^x - x) \, dx \approx 901.393$