NEWTON'S LAW OF COOLING – DAY 1 – HOMEWORK
YOU WANT TO DO THIS HW!!!!!!!

REVIEW PROBLEMS – BE SURE TO LOOK OVER THE PROBLEMS IN YOUR TEXT ON
EXPONENTIAL GROWTH/DECAY AND NEWTON'S LAW OF COOLING!

REMEMBER TO BE NEAT AND USE PROPER NOTATION! PLEASE DO NOT TRY TO SHOW YOUR
WORK ON THIS PAGE!!!

1. Kevin finds a colony of ants in the bottom of his locker while looking for his missing gym shoes. Never wanting to lose an opportunity to learn, he begins spraying the colony with his deodorant to watch them writhe in agony. Assume the rate at which the population decreases is directly proportional to \( A(t) - 10 \), where \( A(t) \) is the amount of ants alive at any time \( t \) (measured in seconds). Kevin notes that there are 1150 ants alive after 10 seconds and 850 after 15 seconds – he’s a quick counter!

   a. Set up an equation that finds the number of ants alive at any time \( t \) – use four decimal places of accuracy when finding constants.
   b. What was the initial population of the colony? The population after 20 seconds? Round both to the nearest ant.
   c. Holly will only look at the colony when there are 10 ants still breathing – how long (rounded to the nearest second) does this take? Show the equation that you are using and solve with your GC!

\[
\frac{dA}{dt} = k(A-10)
\]

\[
\int \frac{1}{A-10} dA = \int k \, dt
\]

\[
\ln(A-10) = kt + M
\]

\[
A-10 = e^{kt+M}
\]

\[
A(t) = 10 + Ce^{kt}
\]

\[
850 = 10 + Ce^{15k}
\]

\[
1150 = 10 + Ce^{10k}
\]

\[
\frac{Ce^{15k}}{Ce^{10k}} = \frac{840}{1140}
\]

\[
e^{5k} = \frac{84}{114}
\]

\[
k = \frac{1}{5} \ln \left( \frac{84}{114} \right)
\]

\[
k \approx -0.0611
\]

\[
C = \frac{840}{e^{15k}} \Rightarrow C \approx 2099.6939
\]

\[
A(t) = 10 + 2099.6939e^{-0.0611t}
\]

b) \( A(0) \approx 2,110 \text{ ants} \)

\( A(20) \approx 629 \text{ ants} \)

c) \( \overline{A(t)} = 10 \)

N.S

Therefore Holly will never look at the ants.
2. The number of “whiffs” (a swing and a miss!) per game pitched that Pedro Martinez of the New York Mets accumulates with his nasty heat can be modeled by the differential equation

\[ \frac{dy}{dx} = \frac{y}{x^2} \]

where \( x \) is the game number that Pedro pitches in and \( y(x) \) is the number of “whiffs” that Pedro induces during that game. By the way, Pedro induces 50 “whiffs” in his opening day start for the Mets.

(a) Find \( y(x) \) from this information!

(b) Store \( y(x) \) and then use this to build a table that shows how many “whiffs” Pedro induces in the first five games of the season and his season total to that point. **Round off to the nearest whiff!**

(c) Pedro’s goal is to induce 130 “whiffs” in one game – could he achieve this goal if he starts 20 games this season? **Explain!**

\[ a) \quad \frac{dy}{dx} = \frac{y}{x^2} \]

\[ \frac{1}{y} \, dy = \frac{1}{x^2} \, dx \]

\[ \int \frac{1}{y} \, dy = \int x^{-2} \, dx \]

\[ \ln y = -x^{-1} + M \]

\[ y(x) = e^{-\frac{1}{x} + M} \]

\[ y(x) = Ce^{-\frac{1}{x}} \]

\[ y(1) = 50 = Ce^{-1} \Rightarrow C = 50e \]

\[ y(x) = 50e \cdot e^{-\frac{1}{x}} \]

\[ y(x) = 50e^{1-\frac{1}{x}} \]

\[ \begin{array}{|c|c|}
\hline
x & y \vphantom{e^x} \\
\hline
0 & 50 \\
1 & 50.426 \\
3 & 97.387 \\
4 & 105.85 \\
5 & 111.28 \\
6 & 115.05 \\
\hline
\end{array} \]

**Total: 446.953 whiffs**

**447 “whiffs”**

**If he only plays 20 games he will not be able to reach his goal of 130 “whiffs.”**
3. The weight of Chris’s buff bod can be modeled by the equation:

\[ w(t) = \frac{155}{1 + 0.25e^{-0.4t}} \]

where \( w(t) \) is Chris’s weight after \( t \) years of rigorous weight training.

a. What is Chris’s initial weight before he starts his training?
b. What is his weight after 1 year, 5 years and 10 years of training?
c. How many years does it take Chris to reach 152 lbs? How about 160 lbs?
d. What weight will Buff Boy max out at?
e. Plot this function using a good y-scale and any other interesting information gotten from the above questions.

\[ w(0) = \frac{155}{1 + \frac{0.25}{4}} = \frac{155}{5/4} = 155 \frac{4}{5} = 31.4 = 124 \text{ lbs} \]

\[ w(1) \approx 132.753 \text{ lbs.} \]
\[ w(5) \approx 149.927 \text{ lbs.} \]
\[ w(10) \approx 154.294 \text{ lbs.} \]

\[ \lim_{t \to \infty} w(t) = 155 \text{ lbs.} \]

c) It takes him about \( 6 \frac{2}{3} \) years to reach 152 lbs, and he never reach 160 lbs.

d) \( \lim_{t \to \infty} w(t) = 155 \text{ lbs.} \)
IN ADDITION, COMPLETE THE FOLLOWING PROBLEMS FROM YOUR BOOK: Page 399-400 #49-56, 59, 60, 71, 72

49. \[-\frac{1}{6} \int_{-6}^{0} x e^{-3x^2} \, dx = -\frac{1}{6} \left( e^{-3 \cdot 0^2} \right) - \frac{1}{6} \left( e^{-3 \cdot (-6)^2} \right) = -\frac{1}{6} \left( \frac{1}{e^3} - 1 \right)\]

50. \[-\int_{\frac{\sqrt{2}}{2}}^{\sqrt{2}} e^{\frac{\sqrt{2} x}{x^2}} \, dx = \int_{\frac{\sqrt{2}}{2}}^{\sqrt{2}} e^{\frac{\sqrt{2} u}{u^2}} \, \frac{dx}{u^2} = \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} e^{\frac{\sqrt{2} u}{u^2}} \, \frac{dx}{u^2} = e^{\frac{\sqrt{2}}{2}} - \sqrt{e}\]

\[u = \sqrt{2} x \quad x = \frac{\sqrt{2}}{2} \Rightarrow u = 2\]
\[\frac{dx}{u^2} = -\frac{1}{u^2} \, dx \quad x = 2 \Rightarrow u = \frac{\sqrt{2}}{2}\]

51. \[\int_{e^2}^{e^x} e^{x^2} + 1 \, dx = \int_{e^2}^{e^x} e^{x^2} + e^{-x^2} \, dx = \frac{1}{3} e^{3x} - e^x - e^{-x} + C\]
\[ S_1 \frac{1}{2} \int \left( \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \right) \, dx = \frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln \left( e^{2x} + e^{-2x} \right) + C \]

\[ u = e^{2x} + e^{-2x} \]
\[ du = (2e^{2x} - 2e^{-2x}) \, dx = 2(e^{2x} - e^{-2x}) \, dx \]

53. \[ \int xe^{-x^2} \, dx = -\frac{1}{2} \int -2x e^{-x^2} \, dx = -\frac{1}{2} e^{-x^2} + C \]

54. \[ \int \frac{e^{x^3}}{x} \, dx = \frac{1}{3} e^{x^3/3} + C \]

55. \[ \int \frac{e^x}{e^{x-1}} \, dx = \int \frac{1}{u} \, du = \ln |u| \bigg|_{e^{-1}}^{e^3} = \ln (e^3) - \ln (e^{-1}) \]

\[ u = e^{x-1}, \quad x=1 \Rightarrow u = e^{-1} \]
\[ du = e^x \, dx, \quad x=3 \Rightarrow u = e^2 \]

56. \[ \int \frac{2e^{2x}}{2(e^{2x} + 1)} \, dx = \frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln |u| \bigg|_{2}^{e^{4+1}} = \frac{1}{2} \left( \ln (e^{4+1}) - \ln (2) \right) \]

\[ u = e^{2x+1}, \quad x=0 \Rightarrow u = 2 \]
\[ du = 2e^{2x} \, dx, \quad x=2 \Rightarrow u = e^{4+1} \]
\[ \int_0^2 \frac{2e^{2x}}{2(e^{2x}+1)} \, dx = \frac{1}{2} \int_2^{e^{4+1}} \frac{1}{u} \, du = \frac{1}{2} \ln u \bigg|_2^{e^{4+1}} = \frac{1}{2} \left( \ln(e^{4+1}) - \ln(2) \right) \]

\( u = e^{2x} + 1 \quad x = 0 \Rightarrow u = 2 \)
\( du = 2e^{2x} \, dx \quad x = 2 \Rightarrow u = e^{4+1} \)

3\( a) \quad y = xe^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 4 \)

\[ A = -\frac{1}{2} \left[ e^{-x^2} \right]_0^4 = -\frac{1}{2} \left( e^{-16} - 1 \right) = \frac{1}{2} - \frac{1}{2e^{16}} \]

6\( a) \quad y = 2e^{-x}, \quad y = 0, \quad x = 0, \quad x = 2 \)

\[ A = -\left[ 2e^{-x} \right]_0^2 = -2 \left( e^{-2} - 1 \right) = 2 \left( 1 - \frac{1}{e^2} \right) \]
\[ (1) \quad \frac{1}{2} \int 2(x+1)^2 \, 5^{(x+1)^2} \, dx = \frac{1}{2} \int 5^u \, du = \frac{1}{2 \ln 5} 5^u + C = \frac{5^{(x+1)^2}}{\ln(25)} + C \]

\[ u = (x+1)^2 \]
\[ du = 2(x+1) \, dx \]

\[ (2) \quad \int \frac{2^{-\frac{1}{2}t}}{t^2} \, dt = \int 2^{-u} \, du = \frac{1}{\ln 2} 2^u + C = \frac{1}{\ln 2} 2^{-\frac{1}{2}t} + C \]

\[ u = -\frac{1}{2}t \]
\[ du = \frac{1}{t^2} \, dt \]